**Module 5: Conic Sections and Polar Coordinates**

**IV. Polar Coordinates**

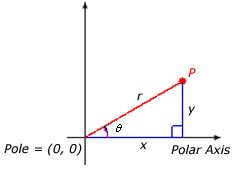
After completing this section, you should be able to:

* plot points and equations in polar coordinates
* convert between rectangular and polar coordinates

So far, points and graphs have been plotted using the (*x*, *y*) rectangular coordinate system. Standard graph paper is printed with a rectangular grid.

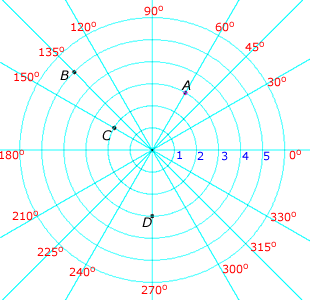
Another system for plotting points and graphs is the **polar coordinate system**. Polar graph paper is printed with concentric circles and radial lines.

In the polar coordinate system, the origin is called the **pole**, and the positive *x*-axis is the **polar axis**. To locate a point *P* in this system, a direction angle *θ* and a value *r* are specified. The angle may be measured in degrees or radians.



To plot a point (*r*, *θ*), first determine the direction. Locate the angle *θ*, measured from the polar axis, with a positive angle measured as usual in the counterclockwise direction and a negative angle in the clockwise direction. If *r* is positive, count off *r* units in the direction specified by the angle. If *r* is negative, count off |*r*| units in the opposite direction.

Consider the points *A*, *B*, *C*, and *D*, which are plotted on the polar graph below.



Can you find polar coordinates for each point?

Note that point *A* is located on the 60° ray. By starting at the center, facing the 60° direction and counting 3 units along the ray, you arrive at the point. So *A* can be represented by (3, 60°) in polar coordinates. However, if you start at the center, face the 240° direction, and count 3 units in the opposite direction, then you will also arrive at point *A*. The point (–3, 240°) also represents *A*. Add or subtract a multiple of 360° and you will get another equivalent representation, such as (3, 420°) or (3, –300°). Unlike the rectangular coordinate system, a point in polar coordinates has infinitely many representations.

Representations of *B* include (5, 135°) and (–5, 315°). A possibility for point *C* is (2, 150°) and for point D, (3, 270°). Angles in the polar coordinate system may also be measured in radians; thus, another possible representation for *D* is (3, 3π/2).

**Conversion between Rectangular Coordinates and Polar Coordinates**

|  |  |
| --- | --- |
| https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/Graphs/polar-coordtes.png | The point (*x*, *y*) in rectangular coordinates corresponds to the point (*r*, *θ*) in polar coordinates, where https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/polar-coord.gif.  https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/cos-eq-y-ovr-r.gif, so *x* = *r* cos *θ.*  https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/sine-eq-y-ovr-r.gif, so *y* = *r* sin *θ.*  https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/tan-eq-y-ovr-r.gif. |

**Example IV.1:** Convert the polar coordinates (–5, π/6) to rectangular coordinates.

Solution:

Given (*r*, *θ*) = (–5, π/6) in polar coordinates, then

*x* = *r* cos *θ* = –5 cos π/6 = https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/mod5-IV-1.gif, and

*y* = *r* sin *θ* = –5 sin π/6 = https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/mod5-IV-1a.gif.

In rectangular coordinates,  https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/mod5-sec-IV-exIV-1.gif.

**Example IV.2:** Convert the rectangular coordinates (–2, 2) to polar coordinates.

Solution:

Given (*x*, *y*) = (–2, 2) in rectangular coordinates, then

https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/mod5-IV-2.gif and

https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/mod5-IV-2a.gif.

Since the point (–2, 2) is in quadrant II and tan *θ* = –1, angle *θ* = 135° (or https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/3pi-ovr-4.gifradians).

In polar coordinates, one possible representation is (*r*, *θ*) = (2https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/sqrt2.gif, 135°).

Another possibility is (*r*, *θ*) = (–2https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M5-Module_5/images/sqrt2.gif, 315°).

An equation in rectangular coordinates involves *x* and/or *y*. An equation in polar coordinates involves *r* and/or *θ*. Equations may be converted from one coordinate system to the other. Some equations are simpler in one system than the other.

**Example IV.3:** Convert the equation *x*2 + *y*2 = 16 to polar coordinates.

Solution:

|  |  |
| --- | --- |
| *x*2 + *y*2 = 16 | Equation of a circle of radius 4 centered at the origin. |
| (*r* cos *θ*)2 + (*r* sin *θ*)2 = 16 | Substitute for *x* and *y*. |
| *r*2 cos2 *θ* + *r*2 sin2 *θ* = 16 | Square each term. |
| *r*2(cos2 *θ* + sin2 *θ*) = 16 | Factor. |
| *r*2 = 16 | Use the identity cos2 *θ* + sin2 *θ* = 1. |
| *r* = 4 | Solve for *r*. |

In polar coordinates, *r* = 4 is the equation of a circle of radius 4 centered at the origin. It is a very simple equation!

The equation *x*2 + *y*2 = 16 is a circle centered at the origin. If the circle is shifted 2 units to the right and 1 unit down, though, the result is a circle with center (2, –1) and the equation becomes (*x* – 2)2 + (*y* + 1)2 = 16. If you convert this equation to polar coordinates, the resulting polar form is not be so simple!

Now try going from polar form to rectangular form.

**Example IV.4:** Convert the polar equation *r* sin *θ* = 5 to rectangular coordinates.

Solution:

Because *y* = *r* sin *θ*, the equation *r* sin *θ* = 5 is simply *y* = 5, a horizontal line.

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